বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - III

Subject: STATISTICS

Paper: C5T

(Linear Algebra and Numerical Analysis)

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt any ***three*** questions from the following : 3×20=60

1. (a) Show that any set of non-null orthogonal vectors are linearly independent. Is the converse true? Justify.

(b) State and prove Gram Schmidt orthogonalization process.

(c) Suppose $\{\underset{\sim}{\alpha}_1, \underset{\sim}{\alpha}_2, \underset{\sim}{\alpha}_3, ..., \underset{\sim}{\alpha}_n\}$ is a set of linearly independent vectors of same order. Examine, if $\{\underset{\sim}{\alpha}_1 + \underset{\sim}{\alpha}_2,\ \underset{\sim}{\alpha}_2 + \underset{\sim}{\alpha}_3, ...,\ \underset{\sim}{\alpha}_n + \underset{\sim}{\alpha}_1\}$ is a set of linearly independent vectors.

(4+2)+8+6=20

2. (a) What is an orthogonal basis of a vector space? Show that the number of vectors in a basis of a vector space is unique?

(b) Suppose $\{\underset{\sim}{\alpha}_1, \underset{\sim}{\alpha}_2, \underset{\sim}{\alpha}_3, \underset{\sim}{\alpha}_4\}$ is a basis of a vector space *V*. Examine, if

$\{\underset{\sim}{\alpha}_1 + \underset{\sim}{\alpha}_2 + \underset{\sim}{\alpha}_3,\ \underset{\sim}{\alpha}_2 + \underset{\sim}{\alpha}_3 + \underset{\sim}{\alpha}_4,\ \underset{\sim}{\alpha}_3 + \underset{\sim}{\alpha}_4 + \underset{\sim}{\alpha}_1,\ \underset{\sim}{\alpha}_4 + \underset{\sim}{\alpha}_1 + \underset{\sim}{\alpha}_2\}$ is also a basis of the vector space *V*.

(c) Define dimension of a vector space. Suppose V_1 is a vector subspace of E_4, where V_1 is defined as

$$V_1 = \left\{ \underset{\sim}{x} : \underset{\sim}{x} = (x_1, x_2, x_3, x_4)';\ x_1 + 2x_2 + 3x_3 + 4x_4 = 0,\ -\infty < x_1, x_2, x_3, x_4 < \infty \right\}.$$

Find a basis of the vector subspace V_1. Also find $\dim(V_1)$. (2+5)+5+(2+5+1)=20

3. (a) Let *A* and *B* be two square matrices of same order *p* such that $A'A = I_p$. Show that $(ABA')^k = AB^k A'$.

(b) Let *A* be an idempotent matrix of order *p*. Show that $(I_p + A)^n = (2^n - 1)A \ \forall n \in N$.

(c) Show that if *A* is an orthogonal matrix, then $(I + A)$ is a singular matrix.

(d) Suppose the inverses of *A* and *B* exits. Show that the inverse of *AB* also exists and $(AB)^{-1} = B^{-1}A^{-1}$.

(e) Let $A = ((a_{ij}))$ be a square matrix of order *p* with $a_{ij} = (a_i - a_j)^2 \ \forall (i, j)$. Find $\det(A)$. 3+3+3+4+7=20

4. (a) Define rank of a square matrix.

(b) Suppose *A* and *B* be two matrices such that *AB* is defined. Show that

$$rank(AB) = \min\{rank(A),\ rank(B)\}.$$

(c) Suppose *A* is a square matrix of order *n* such that $rank(A) = n - 1$. Show that $rank(adj\ A) = 1$.

(d) Let R be a square matrix of order p defined as

$$R = \begin{pmatrix} 1 & r & r & \cdots & r \\ r & 1 & r & \cdots & r \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ r & r & r & \cdots & 1 \end{pmatrix}$$

Find R^{-1}. 2+6+5+7=20

5. (a) Determine the conditions for which the system of equations

$$x_1 + 2x_2 + x_3 = 1$$
$$2x_1 + x_2 + 3x_3 = b$$
$$x_1 + ax_2 + 3x_3 = 1 + b$$

has (i) only one solution, (ii) no solution and (iii) many solution.

(b) Let $T : R^3 \to R^3$ defined by $T(a_1, a_2, a_3) = (2a_2 + a_3, -a_1 + 4a_2 + 5a_3, a_1 + a_3)$. Show that T is a linear mapping and find the matrix of T relative to an orthogonal basis in R^3.

(c) If λ is a characteristic root of a matrix A, then show that λ^m is a characteristic root of the matrix A^m, where m is a positive integer. 10+6+4=20

6. (a) In interpolation of a function $y = f(x)$ corresponding to some x based on $(n + 1)$ pairs of values of x and y, $\{(x_i, y_i), i = 1, 2, ..., n\}$, show that the sum of the coefficients of $y_0, y_1, y_2, ..., y_n$ in Lagrange's interpolation formula is unity.

(b) Obtain the condition for convergence of successive approximations of the real root of the equation $f(x) = 0$ in Newton-Raphson method.

(c) If $P_n(x)$ is a polynomial in x of degree n, show that its rth order finite difference $(r < n)$ is a polynomial in x of degree $(n - r)$. Also show that nth order finite difference of $P_n(x)$ is constant. 6+6+8=20