বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2020

(Under CBCS Pattern)

Semester - III

Subject: MATHEAMATICS

Paper: SEC1T

Full Marks : 40

Time : 2 Hours

Candiates are required to give their answer in their own words as far as practicable.
The figures in the margin indicate full marks.

THEORY OF EQUATIONS

Answer any ***two*** from the following questions : 2×20=40

1. (a) If $\left(x^3+3px+q\right)$ has factor of the form $(x-a)^2$, then show that $q^2+4p^3=0$. 2

(b) Show that the equation of the form $\frac{x^4}{4!}+\frac{x^3}{3!}+\frac{x^2}{2!}+x+1=0$ cannot have a multiple root. 2

(c) Show that the equation $(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0$,

where a, b, c, d are positive and not all equal, has only one real root. 6

(d) Solve the equation $6x^6-25x^5+31x^4-31x^2+25x-6=0\cdot$ 5

(e) If α,β,γ be the roots of the equation $x^3+qx+r=0(r\neq 0)$, find the equation whose roots are $\frac{\beta}{\gamma}+\frac{\gamma}{\beta},\ \frac{\gamma}{\alpha}+\frac{\alpha}{\gamma},\ \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$. 5

2. (a) If α,β,γ be roots of the equation $x^3+px^2+qx+r=0$, then find, in terms of p,q,r the value of $\sum\frac{1}{\alpha^2\beta}\cdot$ 2

(b) Porve that for any real roots a of the equation $x^3-1=0$, ia is a root of the equation $x^{12}-1=0\cdot$ 2

(c) If α, β, γ be the roots are the cubic $x^3-9x+9=0$, then show that

$(\alpha-\beta)(\beta-\gamma)(\gamma-a)=\neq 27$ 6

(d) Solve $x^3-6x-9=0$, by Cardan's method. 5

(e) Solve the equation $x^7-1=0\cdot$ Deduce that $2\cos\frac{2\pi}{7}$, $2\cos\frac{4\pi}{7}$, $2\cos\frac{8\pi}{7}$ are the roots of the equation $t^3+t^2-2t-1=0\cdot$ 5

3. (a) Find the condition that the equation $x^3+px^2+qx+r=0$ may have two roots equal but opposite in sign. 2

(b) Use Descarte's Rule to discuss the nature of roots of the equation $x^4+qx^2+rx-t-0$, if q, r, t are positive numbers. 2

(c) Prove that $x^n-nqx+(n-1)r=0$ has a pair of equal roots if $q^n=r^{n-1}$ and discuss the nature of roots of this equation. 6

(d) Solve by Ferrari's method the equation $x^4-18x^2+32x-15=0\cdot$ 5

(e) Transform the equation $x^3-3x^2+12x+16=0$ into standard form and solve by Cardan's method. 5

4. (a) Find the number and position of the real roots of $x^3+x^2-2x-1=0\cdot$ 7

(b) Form an equation whose roots are less by 2 than the roots of the equation $x^3-5x^2+3x-5=0\cdot$ 7

(c) Show that, a reciprocal equation of standard form can always be depressed to another of half of the roots of this equation. 2

(d) If α, β, γ are the roots of the equation $x^3+3x+1=0$ find the equation whose roots are $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}, \frac{\beta}{\gamma}+\frac{\gamma}{\beta}$ and $\frac{\gamma}{\alpha}+\frac{\beta}{\gamma}$ and hence find the value of $\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$.

(e) Reduce the reciprocal equation $x^5+x^4+x^3+x^2+x+1=0$ to its standard form and then solve it.

LOGIC AND SETS

Answer any ***two*** from the following questions : 2×20=40

1. (a) Write the negation of the following statement : 2

   (i) $n+3\geq 2$ for all $n\in N$.

   (ii) Every complex number is a real number.

   (b) What do you mean Conjuction and Disjuction ? 2

   (c) What do you mean by Principle Conjunctive normal form ? Write complete CNF of 2 variables. Write the function. 5

   $$f(p,q,r)=p\vee(p\wedge q\wedge r)\vee(p\wedge\sim q\wedge\sim r)\vee(q\wedge r)\vee(\sim q\wedge r).$$

   (d) Prove that $\sqrt{2}$ is irrational by contradiction method. 5

   (e) Prove that $(\sim q\wedge q)\rightarrow(\sim p\vee(\sim p\vee q)=\sim p\vee q$ without using truth table. 6

2. (a) Prove that $A-B=A\cap B'$. 2

   (b) Let S be a set containing three elements. How many different binary relations can be defined on S ? 2

(c) Let $D = \{1, 2, 3...9\}$. Determine the truth value of the following statements. 5

(i) $(\forall x \in D) x + 4 < 15$

(ii) $(\forall x \in D) x + 4 > 15$

(iii) $(\forall x \in D) x + 4 \leq 10$

(iv) $(\exists x \in D) x + 4 > 15$

(d) Prove that equivalent using truth table 6

(i) $P \to (q \to r) \equiv (p \wedge q) \to r$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$

(e) Prove that if a finite set S has n elements, then its power set $P(S)$ has 2^n elements. 5

3. (a) Give an example of anti-symmetric relation. 2

(b) Define domain and range of a relation. 2

(c) For the sets $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 6, 9\}$, verify the distributive laws. 5

(d) What is the partial order relation ? Explain with two examples. 5

(e) Give an example of a relation on a set which is reflexive and symmetric but not transitive. 6

4. (a) Let A_0, A_1 and A_2 be three subsets of Z defined by $A_i = \{3n + i : n \in Z\}$ for $i = 0, 1, 2$. Show that A_0, A_1 and A_2 form a partition of the set *Z*. 7

(b) Determine the nature of the following relation R on the set *Z* : *aRb* if and only if *a* – *b* is divisible by 5. 7

(c) Let $A_n = \{x : x \text{ is a multiple of } n, n \in N\}$, Find $A_2 \cup A_7$ and $A_4 \cup A_6$. 2+4

BOOLEAN ALGEBRA

Answer any ***two*** from the following questions : 2×20=40

1. (a) Show that in Boolean algebra *B*, the complement of each element is unique.

(b) Let $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$, the set of all positive divisors of 30. Prove that *B* forms a Boolean algebra with respect to following compositions :

$a + b =$ the l.c.m. of a and b;

$a.b =$ the g.c.d. of and a and b

and $a' = \frac{30}{a}$ for $a, b \in B$

(c) Prove that in a Boolean algebra $(B, +, ., ')$, + is associative . 5+10+5

2. (a) What is the concept of partial ordered relation ? Explain with two examples.

(b) Design a simple circuit connecting two wall switches and a light bulb in such a way that either switch can be used to controll the light independently.

(c) If $(A,\leq)$, and $(B,\leq)$ be two partially order sets then prove that $(A\times B,\leq)$ is partially order set with partial order $'\leq'$ defined by $(a, b)\leq(a_1, b_1)$ if $a \leq a_1$ in *A* and $b \leq b_1$ in *B*. 6+8+6

3. (a) Define modular and distributive lattice with two examples.

(b) Prove that in a Boolean algebra B, $a+b=b$ implies $a.b=a$ and conversely.

(c) Prove that a lattice L is a modular if and only if

$$x \leq y,\quad a \wedge x = a \wedge y,\ a \vee x = a \vee y$$

Implies that $x = y$.

(d) Prove that every chain is a distributive lattice. 6+4+5+5

4. (a) Discuss the concept of sub lattice and lattice homomorphism with an example.

(b) Prove that the set D_{12} of all factors of 12 under divisibility forms a lattice.

(c) Draw the Hasse diagram of the poset $(P(S), \subseteq)$, where $P(S)$ denotes the power set of $S = \{1, 2, 3\}$.

(d) Draw switching circuit which realize the Boolean expression :

$x'yz + x'yz' + xyz$ 6+6+5+3